

Short-time decoherence of Josephson charge qubits in Ohmic and $1/f$ noise environment

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Abstract

In this paper we investigate the short-time decoherence from Ohmic and $1/f$ noise of single Josephson charge qubit (JCQ). At first, we use the short-time approximation to obtain the dynamics of the open JCQ. Then we calculate the decoherence the measure of which is chosen as the maximum norm of the deviation density operator. It is shown that the decoherence from $1/f$ noise plays the central role. The total decoherence from Ohmic and $1/f$ noise is serious at present experiential conditions according to the DiVincenzo criterion.

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Keywords: Short-time decoherence; Ohmic noise; $1/f$ noise

1 Introduction

Quantum bit (qubit) is a key block for building quantum computers. Among various realizations of the qubit for quantum computations that based on the Josephson junctions are considered to be particularly promising candidates because of their scalability, established fabrication techniques, and flexibility in designs. Many kinds of the superconducting Josephson-junction-qubit models are proposed in last years. They are Josephson charge [1], flux [2], phase

[3] and hybridized [4] qubits. According to DiVincenzo [5] proposal, the qubit for quantum computation must satisfy five criteria one of which is the low decoherence criterion. An approximate benchmark of the criterion is a fidelity loss no more than $\sim 10^{-4}$ per elementary quantum gate operation. The decoherence of the Josephson single qubit and coupling qubits has been investigated widely in last years [6]. In these researches the dynamics of the qubits interacting with their environment has been treated by the perturbation method, the path integral method [7], and others [8]. In the researches in general some approximation scheme must be used. The most familiar and frequently used one is the Markov approximation [9].

It has been pointed that the Markov approximation can not be used in low temperature and for short cycle times of quantum computation [10]. However, the qubits based on the Josephson superconducting junction should be worked in low temperature, such as few 10 mK. So the Markov approximation can not be used in the investigations of decoherence for Josephson qubits. Fortunately, a short-time approximation scheme [11] being fit for the investigations has been developed recently by Privman *et al.* [12, 13]. The approaches of V. Privman and his co-worker are rather formal and universal. It is also very interesting to investigate the decoherence of a concrete and physical qubit model because it can help us to know that whether the model satisfy the DiVincenzo low decoherence criterion or not. If the model can not

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satisfy the criterion, the qubit has then not qualification for being a quantum computation hardware.

In this paper we shall investigate that if the JCQ model satisfy the DiVincenzo low decoherence criterion. The same problem has been investigated in our previous works [14, 15] where we suppose that there is only the Ohmic noise in the environment of JCQ. However, recent several experiments (see [16] and within) with Josephson junction circuits have revealed at low frequencies the presence of $1/f$ noise and it is shown that the $1/f$ noise plays a major role in destructing the coherence of qubit in solid-state systems. In this paper, we shall carefully estimate the decoherence of the JCQ in Ohmic and $1/f$ noise environment. Here, the measure of decoherence is based on a standard operator norm $\|A\|$ in the theory of linear operators [17]. The short-time approximation of the split-operator will be used in our investigation.

2 Open JCQ model

The single JCQ Hamiltonian is [18, 19]

$$H_s = 4E_c(n - n_g)^2 - E_J \cos \varphi. \quad (1)$$

Here, $E_c = e^2/2(C_g + C_J)$ is the charging energy; $E_J = I_c \hbar/2e$ is the Josephson coupling energy [20], where I_c is the critical current of the Josephson junction, \hbar the Planck's constant divided 2π , and e the charge of electron; $n_g = C_g V_g/2e = Q_g/2e$ is the dimensionless gate charge, where C_g is the gate capacitance, V_g the controllable gate voltage. The number operator n of (excess) Cooper pair on the island, and the phase operator φ of the superconducting order parameter, are quantum mechanically conjugate [19]. The unavoidable noise of the environment may lead to the dissipation. In general, one takes into account that the environment itself is also a quantum system with a large number degrees of freedom. Usually it is modeled by a bath with a large set of harmonic oscillators [21, 22]

$$H_B = \sum_{\alpha} \left[\frac{1}{2m_{\alpha}} p_{\alpha}^2 + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 x_{\alpha}^2 \right], \quad (2)$$

each of which interacts weakly with the system of interest. The whole Hamiltonian of the system-bath

is [23] then

$$H = H_s + \sum_{\alpha} \left[\frac{1}{2m_{\alpha}} p_{\alpha}^2 + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 \left(x_{\alpha} - \frac{\lambda_{\alpha}}{m_{\alpha} \omega_{\alpha}^2} 2en \frac{C_t}{C_J} \right)^2 \right]. \quad (3)$$

Here, the coupling operator of the environment is $X = \sum \lambda_{\alpha} x_{\alpha}$. If the Josephson coupling energy E_J is much smaller than the charging energy E_c , and both of them are much smaller than the superconducting energy gap Δ , the Hamiltonian H_s of Josephson junction can be parameterized by the number of the Cooper pairs n on the island. When the temperature T is low enough the system can be reduced to a two-state system (qubit) because all other charge states have much higher energy and can be neglected. So the Hamiltonian of the system can approximately reads $H_s = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} B_x \sigma_x$, where $B_z = E_{ch}(1 - 2n_g)$ and $B_x = E_J$. If we choose the working point to make n_g half-integer, say $n_g = 1/2$, we can obtain that

$$H \equiv H_s + H_I + H_B. \quad (4)$$

Here,

$$\begin{aligned} H_s &= -\frac{1}{2} B_x \sigma_x, \quad H_B = \sum_{\alpha} \hbar \omega_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}, \\ H_I &= -\sigma_z \sqrt{\hbar} \sum g_{\alpha} (a_{\alpha}^{\dagger} + a_{\alpha}), \end{aligned} \quad (5)$$

where

$$g_{\alpha} = \lambda_{\alpha} \sqrt{\frac{1}{m_{\alpha} \omega_{\alpha}}} \left(2e \frac{C_t}{C_J} \right). \quad (6)$$

3 Ohmic and $1/f$ noise

Recent studies, both the experimental and theoretical, suggest that the serious noise sources in Josephson devices include two kinds of fluctuations one of which with ohmic spectrum the other with $1/f$ spectrum [16]. In the following, we introduce the spectral densities of the Ohmic and $1/f$ noise. According to [21], from Eq. (3), we can express the spectral density

of the noise as

$$\begin{aligned}
J_X(\omega) &= \frac{\pi}{2} \sum_{\alpha} \left(2e \frac{C_t}{C_J} \right)^2 \frac{|\lambda_{\alpha}|^2}{m_{\alpha} \omega_{\alpha}} \delta(\omega - \omega_{\alpha}) \\
&= \frac{\pi}{2} \hbar \sum_{\alpha} |g_{\alpha}|^2 \delta(\omega - \omega_{\alpha}) \\
&= \frac{\pi}{2} \hbar \int_{-\infty}^{\infty} |g(\omega_{\alpha})|^2 \delta(\omega - \omega_{\alpha}) \\
&= \frac{\pi}{2} \hbar D(\omega) |g(\omega)|^2,
\end{aligned} \tag{7}$$

where $D(\omega)$ is the density of the states of environment modes. On the other hand, the spectral density is related to the power spectrum as [21, 23]

$$S_X(\omega) = J_X(\omega) \hbar \coth(\omega\beta/2). \tag{8}$$

Here, $\beta = \hbar/k_B T$, where k_B is the Boltzmann constant. It has been stressed that the relationship (8) are satisfied not only for the linear coupling model but also for the nonlinear one. To the Ohmic noise case, from the fluctuation-dissipation theorem one has [23]

$$\begin{aligned}
S_X(\omega)|_O &= \left(\frac{2eC_t}{C_J} \right)^2 S_V(\omega)|_O \\
&= \left(\frac{2eC_t}{C_J} \right)^2 \text{Re } Z_t(\omega) \omega \hbar \coth(\omega\beta/2).
\end{aligned} \tag{9}$$

Then, setting $\text{Re } Z_t(\omega) \approx R$, one has

$$J_X(\omega)|_O = (2e)^2 \left(\frac{C_t}{C_J} \right)^2 R \omega. \tag{10}$$

So by using Eq. (7) we have

$$D(\omega) g^2(\omega)|_O = 4 \frac{(2e)^2}{\hbar} \left(\frac{C_t}{C_J} \right)^2 R \omega. \tag{11}$$

Considering the cutoff of the frequency ω we can set

$$D(\omega) g^2(\omega)|_O = \eta \omega \exp\left(\frac{\omega}{\omega_c}\right). \tag{12}$$

Here, ω_c is the cutoff frequency and $\eta = 4 \frac{R}{R_Q} \left(\frac{C_t}{C_J} \right)^2$, where $R_Q = h/(2e)^2 \approx 6.5 \text{ k}\Omega$ [24]. In the model of

the JCQ circuit the typical impedance of the control line is $R \approx 50 \Omega$, and $C_g \approx 10^{-18} \text{ F}$, $C_J \approx 10^{-16} \text{ F}$, so we can obtain $\eta \approx 10^{-6}$.

The $1/f$ noise is considered deriving from the background charge fluctuations in the circuits. It can be expressed an effective noise of the gate charge, i. e., $S_{Q_g}(\omega) = \alpha_f e^2/\omega$. Recent experiments proposed at relevant temperatures $\alpha_f \sim 10^{-7} - 10^{-6}$. According to Ref. [23] this noise can be translated into the fluctuations of X , namely,

$$S_X(\omega)|_f = E_f^2/\omega, \tag{13}$$

where $E_f = 4E_c \sqrt{\alpha_f}$. Comparing the Eq. (13) with Eq. (8) we have

$$J_X(\omega)|_f = \frac{16E_c^2 \alpha_f}{\hbar \omega \coth(\omega\beta/2)}. \tag{14}$$

So

$$\begin{aligned}
D(\omega) g^2(\omega)|_f &= \frac{32E_c^2 \alpha_f}{\pi \hbar^2 \omega \coth(\omega\beta/2)} \\
&= \frac{\kappa \alpha_f}{\omega \coth(\omega\beta/2)},
\end{aligned} \tag{15}$$

where

$$\kappa = \frac{64E_c^2}{h\hbar} \approx 1.5 \times 10^{25}. \tag{16}$$

In fact, in a JCQ circuit these two kinds of noise are existed at the same time. So the spectral density of the total noise is

$$J(\omega) = J_X(\omega)|_O + J_X(\omega)|_f. \tag{17}$$

So we have

$$D(\omega) g^2(\omega) = D(\omega) g^2(\omega)|_O + D(\omega) g^2(\omega)|_f. \tag{18}$$

In the following, we shall use above three kinds of different noise of the environment investigating the decoherence of the JCQ.

4 Short-time dynamics of JCQ

Before studying the decoherence we shall investigate the dynamics of the open JCQ with the short-time

approximation. Suppose the initial state of the JCQ-bath be $R(0) = \rho(0) \otimes \Theta$, where $\rho(0)$ is the initial state of JCQ and Θ is the initial state of the environment. We set Θ is the product of the bath modes density matrices θ_k . In the initial states, each bath mode k is assumed to be thermalized, namely,

$$\theta_k = \frac{e^{-\beta M_k}}{\text{Tr}_k(e^{-\beta M_k})}, \quad (19)$$

where $M_k = \omega_k a_k^\dagger a_k$. The evolution operator of the JCQ-bath is then

$$U = e^{-iH\tau/\hbar} = e^{-i(H_s + H_I + H_B)\tau/\hbar}. \quad (20)$$

Due to non-conservation of H_s in this system, the evolution operator cannot be in a general way expressed as $e^{-iH_s\tau/\hbar}e^{-i(H_I + H_B)\tau/\hbar}$. But in the short-time approximation, the operator can be approximately expressed as [25, 26]

$$U = e^{-iH_s\tau/2\hbar}e^{-i(H_I + H_B)\tau/\hbar}e^{-iH_s\tau/2\hbar} + o(\tau^3). \quad (21)$$

It has been proved that the expression is accurate enough for the time being short to the characteristic time. In the following we only investigate the case that the system evolve within the time $\tau < 10$ ps, the characteristic time of the single JCQ is $\tau = 12.7$ ps (when $E_J = 51.8 \mu\text{eV}$) [15]. So the elements of the reduced density matrix $\rho(\tau)$ in the basis of operator H_s can be expressed as

$$\rho_{mn} = \text{Tr}_B \langle \varphi_m | e^{-iH_s\tau/2\hbar} e^{-i(H_I + H_B)\tau/\hbar} e^{-iH_s\tau/2\hbar} R(0) e^{iH_s\tau/2\hbar} e^{i(H_I + H_B)\tau/\hbar} e^{iH_s\tau/2\hbar} | \varphi_n \rangle, \quad (22)$$

where $\{m, n\} = 0$ or 1 . The ρ is a matrix with 2 by 2. In the following we set $t = \tau/\hbar$. Through some calculations we can obtain the evolution of the density matrix elements $\rho_{10}(t)$ and $\rho_{11}(t)$ as [14, 15]

$$\begin{aligned} \rho_{10}(t) &= \frac{1}{2}\rho_{10} \left(1 - e^{-B^2(t)} + e^{itE_J} + e^{itE_J - B^2(t)} \right), \\ \rho_{11}(t) &= \frac{1}{2}\rho_{00} \left(1 - e^{-B^2(t)} \right) + \frac{1}{2}\rho_{11} \left(1 + e^{-B^2(t)} \right), \end{aligned} \quad (23)$$

where $\rho_{00} = \rho_{00}(0)$, $\rho_{11} = \rho_{11}(0)$, $\rho_{10} = \rho_{10}(0)$, and

$$\begin{aligned} B^2(t) &= 8 \sum_k \frac{|g_k|^2}{\omega_k^2} \sin^2 \frac{\omega_k t}{2} \coth \frac{\beta \omega_k}{2}, \\ C(t) &= \sum_k \frac{|g_k|^2}{\omega_k^2} (\omega_k t - \sin \omega_k t). \end{aligned} \quad (24)$$

When the summation in Eq. (24) is converted to integration in the limit of infinite number of the bath modes, one has

$$B^2(t) = 8 \int d\omega D(\omega) g(\omega)^2 \omega^{-2} \sin^2 \frac{\omega t}{2} \coth \frac{\beta \omega}{2}, \quad (25)$$

for the real $g(\omega)$. In fact this $B^2(t)$ is the mean-squared value of the magnitude of the phase noise in Ref. [27]. From the deriving of $B^2(t)$ we know that it results from the choice of bath model and the coupling forms of JCQ-bath rather than the choice of the approximation scheme. But the evolution of the matrix elements as Eqs. (23) is resulted from the short-time approximation and it can not be obtained by using the Markov approximation and others. By using above results we can yield a good quantitative estimation to the decoherence behavior of JCQ in a short time.

5 Decoherence of JCQ

It is shown that the measure

$$D(t) = \sup_{\rho(0)} (\|\sigma(t, \rho(0))\|_\lambda) \quad (26)$$

is suitable for estimating the decoherence of the qubit gates [12, 13]. Here, the norm $\|\sigma\|_\lambda$ is defined as

$$\|\sigma\|_\lambda = \sup_{\varphi \neq 0} \left(\frac{\langle \varphi | \sigma | \varphi \rangle}{\langle \varphi | \varphi \rangle} \right)^{\frac{1}{2}}. \quad (27)$$

For a qubit, it is

$$\|\sigma\|_\lambda = \sqrt{|\sigma_{10}|^2 + |\sigma_{11}|^2}. \quad (28)$$

Here, the deviation operator σ is defined as

$$\sigma(\tau) = \rho(\tau) - \rho^i(\tau), \quad (29)$$

where $\rho(\tau)$ and $\rho^i(\tau)$ are density matrixes of the “real” evolution (with interaction) and the “ideal” one (without interaction) of the investigated system. The evolution of the closed JCQ is $\rho_{11}^i(t) = \rho_{11}$, and $\rho_{10}^i(t) = \rho_{10}e^{itE_J}$. So for the open JCQ we have

$$\begin{aligned}\sigma_{10}(t) &= \frac{1}{2}\rho_{10}\left(1 - e^{-B^2(t)}\right)\left(1 - e^{itE_J}\right), \\ \sigma_{11}(t) &= \frac{1}{2}\left(1 - e^{-B^2(t)}\right)(\rho_{00} - \rho_{11}).\end{aligned}\quad (30)$$

Thus, we have

$$\begin{aligned}\|\sigma(t)\|_\lambda &= \frac{1}{2}\left(1 - e^{-B^2(t)}\right) \\ &\quad \left\{(\rho_{00} - \rho_{11})^2 + 4|\rho_{10}|^2 \sin^2 \frac{E_J t}{2}\right\}^{\frac{1}{2}}.\end{aligned}\quad (31)$$

From Eq.(31) we know that a pure state

$$\rho_1(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ or } \rho_2(0) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (32)$$

will make the $\|\sigma(t)\|_\lambda$ into $D(t)$. In the following calculations we use the initial state $\rho_1(0)$ and choose the Josephson energy $E_J = 51.8 \mu\text{eV}$ according to Ref. [1].

Fig.1

Fig.2

Fig.3

By using Eqs. (11,15,18), (25), and (31) we can plot the decoherence of JCQ versus time t . In Fig.1 we plot the decoherence from Ohmic noise of the JCQ versus time t . This decoherence is mainly derived from the higher frequency parts of the Ohmic

noise. Here, we confine the frequencies of bath modes in $1 \text{ GHz} \sim 50 \text{ GHz}$ this is a very broad band of frequencies of the environment noise. The lowest line in Fig.1 plots the decoherence at temperature $T_1 = 0.0300 \text{ K}$. It is shown that when $t < 0.1$, namely, $\tau < 6.582 \times 10^{-11} \text{ s}$ the decoherence is much smaller than DiVincenzo criterion 10^{-4} . The middle and the upmost lines plot the decoherence at temperature $T_2 = 0.1500 \text{ K}$ and $T_3 = 0.1875 \text{ K}$. From the plots we see that the decoherence will increase with the increasing of the temperature T but it cannot increase to the maximum value of the DiVincenzo criterion permitting at $T = 0.1875 \text{ K}$ within $t = 0.02$ ($\tau \approx 12.7 \text{ ps}$). Fig.2 shows the decoherence from $1/f$ noise of the JCQ in different α_f . It is shown that this decoherence mainly derived from the low frequency parts of the $1/f$ noise and it is sensitive to the changing of α_f . In the calculations, we take the frequencies of the environment modes from 1 kHz to 1 GHz the numerical simulation shows that the spectrum is broad enough. In the Fig.2 the lowest, middle, and upmost lines plot the decoherence at $\alpha_f = 1.0 \times 10^{-7}$, 1.1×10^{-7} and 1.2×10^{-7} . A further calculation shows that if only $\alpha_f > 5.0 \times 10^{-8}$ the decoherence will affect the JCQ as the qubit for making quantum computers because the decoherence is bigger than the DiVincenzo low decoherence criterion in the single operation time. So we should decrease the α_f in order to make JCQ be a qubit for quantum computation. In Fig.3 we simulate three kinds of cases where both the Ohmic and $1/f$ noise are considered. Here, we set $T = 30 \text{ mK}$ and $\alpha_f = 5 \times 10^{-8}$ (the lowest line), $\alpha_f = 4 \times 10^{-8}$ (the middle line) and $\alpha_f = 3 \times 10^{-8}$ (the upmost line). The frequencies of bath modes is set in $1 \text{ kHz} \sim 1 \text{ GHz}$ for $1/f$ noise and in $1 \text{ GHz} \sim 50 \text{ GHz}$ for Ohmic noise. It is shown that when the α_f do not exceed 5×10^{-8} , within $\tau = 12.7 \text{ ps}$ the decoherence from the Ohmic noise and $1/f$ noise is endurable.

6 Conclusions

In this paper we investigated the short-time decoherence results from the Ohmic and $1/f$ noise of the JCQ. It is shown that the decoherence from $1/f$ noise

is larger than that from Ohmic noise to the JCQ model. To the Ohmic noise the higher frequency parts play a major role to the decoherence. We take a larger range of the frequencies of the environment modes in our numerical simulation than usually proposed. It is shown that in usually experimental temperature this decoherence is not serious comparing to the DiVincenzo criterion. Unlike the Ohmic noise case, to the $1/f$ noise the decoherence is mainly determined by the lower frequency parts. It is shown that when $\alpha_f \lesssim 5 \times 10^{-8}$, within $\tau = 12.7$ ps this decoherence is also not serious according to the DiVincenzo criterion. However, the present experiments show that the value of α_f is between 10^{-7} and 10^{-6} . Thus, if we wish to make the JCQ become a optimal qubit model we should decrease the value of α_f . The value of α_f is related to the temperature. It may decrease with the decreasing of temperature. Carefully finding out the correlation of the value of α_f and other parameters (include the temperature) is a important task for our investigating the decoherence of JCQ model in the future.

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7 Figs. captions

Fig.1 The decoherence from Ohmic noise within short time. The lowest, middle and upmost lines correspond to $T_2 = 0.0300 \text{ K}$, $T_2 = 0.1500 \text{ K}$ and $T_3 = 0.1875 \text{ K}$. Here, the frequencies of bath modes is set in 1 GHz \sim 50 GHz.

Fig.2 The decoherence from $1/f$ noise within short time. The lowest, middle and upmost lines correspond to $\alpha_f = 1.0 \times 10^{-7}$, $\alpha_f = 1.1 \times 10^{-7}$, and $\alpha_f = 1.3 \times 10^{-7}$. Here, the frequencies of bath modes is set in 1 kHz \sim 1 GHz.

Fig.3 The decoherence from Ohmic noise and $1/f$ noise within short time. The lowest, middle and upmost lines correspond to $\alpha_f = 3 \times 10^{-8}$, $\alpha_f = 4 \times 10^{-8}$, and $\alpha_f = 5 \times 10^{-8}$. Here, the temperature is set $T = 30 \text{ mK}$, the frequencies of bath modes is





